Medium Effects on Binary Collisions with the Δ Resonance

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Abstract

To facilitate the relativistic heavy-ion calculations based on transport equations, the binary collisions involving a Δ resonance in either the entrance channel or the exit channel are investigated within a Hamiltonian formulation of πNN interactions. An averaging procedure is developed to define a quasi-particle Δ^* and to express the experimentally measured $NN \to \pi NN$ cross section in terms of an effective $NN \to N\Delta^*$ cross section. In contrast to previous works, the main feature of the present approach is that the mass and the momentum of the produced Δ^*'s are calculated dynamically from the bare $\Delta \leftrightarrow \pi N$ vertex interaction of the model Hamiltonian and are constrained by the unitarity condition. The procedure is then extended to define the effective cross sections for the experimentally inaccessible $N\Delta^* \to NN$ and $N\Delta^* \to N\Delta^*$ reactions. The predicted cross sections are significantly different from what are commonly assumed in relativistic heavy-ion calculations. The Δ potential in nuclear matter has been calculated by using a Bruckner-Hartree-Fock approximation. By including the mean-field effects on the Δ propagation, the effective cross sections of the $NN \to N\Delta^*$, $N\Delta^* \to NN$ and $N\Delta^* \to N\Delta^*$ reactions in nuclear matter are predicted. It is demonstrated that the density dependence is most dramatic in the energy region close to the pion production threshold.

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I. INTRODUCTION

Nuclear matter with a high population of nucleon resonances (Δ, N^*'s) is expected to exist in some astrophysical objects and can be created in relativistic heavy-ion collisions. With the very dedicated experimental efforts at GSI, AGS and CERN in the past decade, it appears that the properties of such a nuclear system can now be investigated. Experimental evidence for creating a Δ-rich nuclear system during relativistic heavy-ion collisions have recently been reported [1,2]. This interesting development has raised two theoretical issues. First, it is necessary to examine the extent to which the theoretical models employed in identifying the Δ-rich matter are valid. In the calculations based on transport equations [3–9], some assumptions were made to define the cross sections of binary collisions involving a Δ in either the entrance channel or the exit channel. This needs to be clarified within a rigorous formulation of the scattering theory with resonance excitations. The medium effects on Δ propagation assumed in those calculations must also be examined from a more fundamental point of view. One possibility is to calculate the Δ mean field using the well-developed nuclear many-body methods [10,14]. The second theoretical issue is how to develop a microscopic approach to understand the Δ-rich matter in terms of the elementary NN, $N\Delta$ and $\Delta\Delta$ collisions and their coupling with the pion production channels. These two theoretical issues are closely related and must be addressed within the same theoretical framework. The heart of the problem is to define precisely and to determine quantitatively all possible binary collisions in the Δ-rich matter. In this paper, we will address this problem by employing a Hamiltonian model of πNN interactions which was previously developed to describe NN scattering up to 2 GeV [11–13] and the Δ-nucleus dynamics in pion-nucleus reactions [14]. Our main objective is to provide the theoretical input to the relativistic heavy-ion calculations using transport equations [3–9].

In section II, we briefly review the coupled $NN \oplus N\Delta$ scattering equations within the πNN Hamiltonian model developed in Refs. [11–13]. We derive formulas in section III to relate the experimentally observed $NN \to \pi NN$ reaction to the resonating $N\Delta$ state. An

averaging procedure is then developed to define a quasi-particle Δ^* and to evaluate the cross sections for binary collisions with a Δ^* in either the entrance channel or the exit channel. In section IV, we recall the approach of Ref. [14] to calculate the mean-field effects on Δ propagation which are then used to calculate the medium-corrected cross sections. In section V, we present our predictions of the cross sections for $NN \rightarrow N\Delta^*$, $N\Delta^* \rightarrow NN$ and $N\Delta^* \rightarrow N\Delta^*$ transitions in free space and in nuclear medium. A summary is given in section VI.

II. HAMILTONIAN MODEL OF πNN INTERACTIONS

An important advance in intermediate energy physics is the development of a microscopic approach to understand the nuclear reactions induced by pions, protons, and electrons in terms of the interactions between π, N, and Δ degrees of freedom. Such an approach must start with a πNN model which can describe the following elementary processes

$$\pi N \rightarrow \pi N, \qquad E_L \leq \ 300 \ \text{MeV} \,; \tag{1}$$

$$NN \rightarrow NN \,,$$

$$\rightarrow \pi NN, \qquad E_L \leq 1000 \ \text{MeV} \,; \tag{2}$$

$$\pi d \rightarrow \pi d \,,$$

$$\rightarrow NN \,,$$

$$\rightarrow \pi NN \qquad E_L \leq \ 300 \ \text{MeV} \,. \tag{3}$$

The developments of πNN models have been reviewed in Ref. [16]. In this work we employ the πNN model developed in Ref. [11–13]. The main feature of the constructed πNN Hamiltonian is that the $NN \leftrightarrow N\Delta$ interaction can be directly constrained by data on the $NN \rightarrow N\Delta \rightarrow \pi NN$ reaction, and the $N\Delta \rightarrow N\Delta$ interaction can also be related to the πd reactions. This allows a more realistic determination of the phenomenological parts of the interactions involving the Δ. For example, it was found [11–13] that the ranges of the πNN and $\pi N\Delta$ vertices in the $NN \leftrightarrow N\Delta$ transition potential must be less than about 750

MeV for a monopole form if the the $NN \to \pi NN$ reaction cross sections can be reasonably described. This phenomenological procedure is not avoidable in any meson-exchange model since a fundamental theory of short-range NN and $N\Delta$ interactions is still not available.

In this first application of the πNN model to relativistic heavy-ion collisions, we will focus on the Δ resonance and neglect the weaker non-resonant πN interactions. The πNN model Hamiltonian within the formulation of Refs. [11–13] can then be written as

$$H = H_0 + H_{int}, \tag{4}$$

where H_0 is the sum of free energy operators for the N, Δ and π degrees of freedom, and

$$H_{int} = \sum_{i=1}^{2} [h_{\pi N,\Delta}(i) + h_{\Delta,\pi N}(i)]$$
$$+ \frac{1}{2} \sum_{i,j=1}^{2} [V_{NN,NN}(i,j) + V_{NN,N\Delta}(i,j) + V_{N\Delta,NN}(i,j) + V_{N\Delta,N\Delta}(i,j)] \tag{5}$$

Note that the vertex interactions $h_{\pi N,\Delta}$ and $h_{\Delta,\pi N}$ describe the $\pi N \leftrightarrow \Delta$ transition. They can renormalize the bare Δ mass and generate a $N\Delta \to \Delta N$ interaction due to the exchange of an "on-mass-shell" pion. In the following presentation, this $N\Delta \to \Delta N$ is included in $V_{N\Delta,N\Delta}$ of Eq. (5).

The derivation of the πNN scattering equations from the above model Hamiltonian can be found in section II of Ref. [12]. By some straightforward derivations using Eqs. (3.27), (3.31), and (3.24) of Ref. [12], we obtain the following equations defined in the coupled $NN \oplus N\Delta$ space

$$T_{NN,NN}(E) = \hat{V}_{NN,NN}(E) + \hat{V}_{NN,NN}(E) G_{NN}(E) T_{NN,NN}(E), \tag{6}$$

$$T_{N\Delta,NN}(E) = \Omega_{N\Delta}^{(-)+}(E) V_{N\Delta,NN} [1 + G_{NN}(E) T_{NN,NN}(E)], \tag{7}$$

$$T_{NN,N\Delta}(E) = [1 + T_{NN,NN}(E) G_{NN}(E)] V_{NN,N\Delta} \Omega_{N\Delta}^{(+)}(E), \tag{8}$$

$$T_{N\Delta,N\Delta}(E) = t_{N\Delta,N\Delta}(E) + T_{N\Delta,NN} V_{NN,N\Delta} \Omega_{N\Delta}^{(+)}(E). \tag{9}$$

In the above equations, we have defined the propagators

$$G_{NN}(E) = \frac{P_{NN}}{E - H_0 + i\epsilon}, \tag{10}$$

$$G_{N\Delta}(E) = \frac{P_{N\Delta}}{E - H_0 - \Sigma(E)}, \tag{11}$$

where the Δ self-energy is determined by the $h_{\pi N \leftrightarrow \Delta}$ vertex interaction in the presence of a spectator nucleon

$$\Sigma_\Delta(E) = \sum_{i=1}^{2} h_{\Delta,\pi N}(i) \frac{P_{\pi NN}}{E - H_0 + i\epsilon} h_{\pi N, \Delta}(i) \,. \tag{12}$$

P_{NN}, $P_{N\Delta}$, and $P_{\pi NN}$ are respectively the projection operators for the NN, $N\Delta$, and πNN states.

The main feature of the above scattering formulation is that the effects due to the $V_{N\Delta,N\Delta}$ interaction are isolated in the $N\Delta$ scattering t-matrix $t_{N\Delta,N\Delta}$ defined by

$$t_{N\Delta,N\Delta}(E) = V_{N\Delta,N\Delta} + V_{N\Delta,N\Delta} G_{N\Delta}(E) t_{N\Delta,N\Delta}(E) \,, \tag{13}$$

and the $N\Delta$ scattering operators defined by

$$\Omega_{N\Delta}^{(+)}(E) = 1 + G_{N\Delta}(E) t_{N\Delta,N\Delta}(E) \,, \tag{14}$$

$$\Omega_{N\Delta}^{(-)+}(E) = 1 + t_{N\Delta,N\Delta}(E) G_{N\Delta}(E) \,. \tag{15}$$

The $N\Delta$ scattering also has a contribution to the effective NN potential in Eq. (6). Explicitly, we have

$$\hat{V}_{NN,NN}(E) = V_{NN,NN} + U_{NN,NN}^{(2)}(E) \,, \tag{16}$$

with

$$U_{NN,NN}^{(2)}(E) = V_{NN,N\Delta} G_{N\Delta}(E) \left[1 + t_{N\Delta,N\Delta}(E) G_{N\Delta}(E) \right] V_{N\Delta,NN} \,. \tag{17}$$

The task is then to find an appropriate parameterization of the model Hamiltonian Eq. (5) to best reproduce the data of the πNN processes listed in Eqs. (1)-(3). This was first achieved by Betz and Lee [17] using the separable parameterization. In this work we consider the meson-exchange parameterization of Refs. [11–13]. It was found that the πNN data can be best reproduced by using: (1) the vertex interaction $h_{\pi N \leftrightarrow \Delta}$ determined from fitting the πN scattering phase shift in P_{33} channel, (2) the one-pion-exchange model [18] of the transition potentials $V_{NN \leftrightarrow N\Delta}$ and $V_{N\Delta,N\Delta}$ with a monopole form factor of a cutoff

parameter $\Lambda = 650$ MeV/c (3), the NN interaction $V_{NN,NN}$ of Eq. (16) is defined by using a subtraction of the Paris potential [19]

$$V_{NN,NN} = V_{Paris} - U^{(2)}_{NN,NN}(E = E_s)\,, \tag{18}$$

with $E_s = 10$ MeV laboratory energy. Note that the above definition of the NN interaction amounts to removing phenomenologically the two-pion-exchange with an intermediate $N\Delta$ state from the Paris potential, in order to avoid the double counting of the $N\Delta$ effect. This approach was also developed independently by Sauer and collaborators [20]. The details of the determination of the model Hamiltonian as well as comparisons with the πNN data can be found in Refs. [11–13]. Here we illustrate the validity of the model by showing in Fig. 1 that the predictions of the model are also in good agreement with the most recent np polarization data [15].

Before we proceed further, it is important to clarify the meaning of the Δ degree of freedom of the considered πNN model. Because of the presence of the vertex interaction $h_{\Delta \leftrightarrow \pi N}$, the Δ is certainly not a physical particle which can be detected experimentally. To investigate the Δ dynamics, we need to consider the $NN \leftrightarrow \pi NN$ and $\pi NN \to \pi NN$ reactions. The amplitudes of these reactions can be calculated from the vertex interaction $h_{\Delta \leftrightarrow \pi N}$ and the baryon-baryon t-matrix defined in Eqs. (6)-(9) by the following equations

$$T_{NN,\pi NN}(E) = T_{NN,N\Delta}(E)G_{N\Delta}\left[\sum_{i=1}^{2} h_{\Delta,\pi N}(i)\right]\,, \tag{19}$$

$$T_{\pi NN,NN}(E) = \left[\sum_{i=1}^{2} h_{\pi N,\Delta}(i)\right] G_{N\Delta}(E)T_{N\Delta,NN}(E)\,, \tag{20}$$

$$T_{\pi NN,\pi NN}(E) = \left[\sum_{i=1}^{2} h_{\pi N,\Delta}(i)\right] G_{N\Delta}(E)T_{N\Delta,N\Delta}(E) \left[\sum_{i=1}^{2} h_{\Delta,\pi N}(i)\right]\,. \tag{21}$$

III. EFFECTIVE BINARY CROSS SECTIONS WITH Δ^*

We will only consider the transport equations of relativistic heavy-ion collisions which are determined by the cross sections of the collisions between "on-mass-shell" physical particles.

Within the model defined by the πNN Hamiltonian Eqs. (4)-(5), the transport equations can then only be written in terms of π and N variables. It is however possible to express the dynamics associated with the pion in terms of a quasi-particle Δ^* which has a mass equal to the invariant mass of a πN subsystem. The transport equations can then be re-written in terms of N and Δ^*. In such a approach, the mass of the Δ^* can range from $m_N + m_\pi$ to the highest energy allowed by the collision energy. To carry out calculations, it is necessary to define the $NN \leftrightarrow N\Delta^*$ and $N\Delta^* \rightarrow N\Delta^*$ cross sections. In the following we will try to define rigorously these cross sections in terms of the $NN \leftrightarrow \pi NN$ and $\pi NN \leftrightarrow \pi NN$ amplitudes defined in the previous section.

To proceed, let us first consider the $NN \rightarrow \pi NN$ and the three-body kinematics in the center of mass frame (Fig. 2):

$$\vec{k} = \vec{k}_1 \,,$$
$$= -(\vec{k}_2 + \vec{k}_3) \,. \tag{22}$$

For a given total collision energy E and a given momentum $\vec{k}$, the invariant mass M and the internal relative momentum $\vec{q}$ of the $\pi N(23)$ subsystem, called Δ^*, are defined by the following relations

$$E = E_N(k) + [M^2 + k^2]^{1/2} \,, \tag{23}$$
$$M = E_N(q) + E_\pi(q) \,, \tag{24}$$

where $E_\alpha(q) = (m_\alpha^2 + q^2)^{1/2}$ is the energy of a particle with mass m_α. Eq. (23) also leads to a useful relation

$$M^2 = E^2 - 2EE_N(k) + m_N^2 \,. \tag{25}$$

Clearly, Eqs. (24) and (25) imply that the invariant mass of the Δ^* is restricted to the region $(m_N + m_\pi) \leq M \leq (E - m_N)$. By using Eqs. (23)-(24), the momenta k and q can be written as

$$k = \frac{1}{2E}[(E^2 - m_N^2 - M^2)^2 - 4M^2 m_N^2]^{1/2}, \tag{26}$$

$$q = \frac{1}{2M}[(M^2 - m_N^2 - m_\pi^2)^2 - 4m_N^2 m_\pi^2]^{1/2}. \tag{27}$$

In terms of the kinematical variables defined above and in Fig. 2, the matrix element of the $NN \to \pi NN$ transition operator, Eq. (20), can be written (suppressing spin-isospin indices) as

$$< \vec{k}, \vec{q} \mid T_{\pi NN, NN}(E) | \vec{k}_0 >= h_{\pi N, \Delta}(\vec{q}) \frac{1}{E - E_N(k) - E_\Delta(k) - \Sigma(E, \vec{k})} T_{N\Delta, NN}(\vec{k}, \vec{k}_0, E). \tag{28}$$

We choose the normalization $< \vec{k}' \mid \vec{k} >= \delta(\vec{k}' - \vec{k})$. The scattering t-matrix is related to the S-matrix by (in operator form)

$$S(E) = 1 - 2\pi i \delta(E - H_0) T(E). \tag{29}$$

We next define the following partial-wave expansions

$$h_{\pi N, \Delta}(\vec{q}) = y_{l_{23} s_{23}}^{s_\Delta m_{s_\Delta}}(\hat{q}) h_{\pi N, \Delta}(q) < s_\Delta m_{s_\Delta} \mid, \tag{30}$$

and

$$T_{N\Delta, NN}(\vec{k}, \vec{k}_0, E) = \sum_{J, M_J} \sum_{LS, L_\Delta S_\Delta} y_{L_\Delta S_\Delta}^{JM_J}(\hat{k}) T_{L_\Delta S_\Delta, LS}^J(k, k_0, E) y_{LS}^{JM_J^+}(\hat{k}_0). \tag{31}$$

In the above equations, we have defined the spin-angular vector

$$y_{ls}^{jm}(\hat{p}) = \sum_{m_l m_s} \sum_{m_{s_1} m_{s_2}} Y_{l m_l}(\hat{p}) \mid s_1 m_{s_1} > \mid s_2 m_{s_2} > \\ \times < l s m_l m_s | j m >< s_1 s_2 m_{s_1} m_{s_2} | s m_s >, \tag{32}$$

where $Y_{l m_l}(\hat{q})$ is the usual spherical harmonic function, (s_i, m_{s_i}) are the spin variables for the i-th particle.

By using the definitions Eqs. (28)-(29) and the variables defined by Eqs. (23)-(27), we can write the total cross section of the two-step process $NN \to N\Delta \to NN\pi$ (Fig. 2) as

$$\sigma^{tot}(E) = \frac{1}{(2s_1' + 1)(2s_2' + 1)} \sum_{spins} \frac{(2\pi)^4}{k_0^2} \left[\frac{k_0 E_N(k_0)}{2} \right] \int d\vec{k} \int d\vec{q}$$

$$\delta(E - E_N(k) - \sqrt{(E_N(q) + E_\pi(q))^2 + k^2}) \mid \sum_{m_{s_\Delta}} < m_{s_2} m_{s_3} \mid h_{\pi N, \Delta}(\vec{q}) \mid m_{s_\Delta} >$$

$$\times \frac{1}{E - E_N(k) - E_\Delta(k) - \Sigma(k, E)} < m_{s_\Delta} m_{s_1} \mid T_{N\Delta, NN}(\vec{k}, \vec{k}_0, E) \mid m_{s_1}' m_{s_2}' >|^2, \tag{33}$$

where the initial momentum k_0 is defined by the total energy $E = E_N(k_0) + E_N(k_0)$. The Δ self-energy is found to be

$$\Sigma(k, E) = \int \frac{|h_{\Delta,\pi N}(q')|^2 q'^2 dq'}{[(E - E_N(k))^2 - k^2]^{1/2} - E_N(q') - E_\pi(q') + i\delta} \,. \tag{34}$$

Substituting the expansions Eqs. (30)-(31) into Eq. (33) and integrating over angles $\hat{q}$ and $\hat{k}$, we obtain

$$\sigma^{tot}_{NN\pi \leftarrow NN}(E) = \sum_{J,LS,L_\Delta S_\Delta} d\sigma^J_{L_\Delta S_\Delta, LS}(E), \tag{35}$$

with

$$d\sigma^J_{L_\Delta S_\Delta, LS}(E) = \frac{4\pi}{k_0^2}\left(J + \frac{1}{2}\right)\int_0^{k_{max}} dk \mid N^{1/2}_{N\Delta}(k) T^J_{L_\Delta S_\Delta, LS}(k, k_0, E) \rho^{1/2}_{NN}(E) \mid^2, \tag{36}$$

where $k_{\max}$ can be calculated from Eq. (26) by setting $M = m_\pi + m_N$, and

$$\rho_{NN}(E) = \pi \frac{k_0 E_N(k_0)}{2} \,, \tag{37}$$

$$N_{N\Delta}(k) = \pi k^2 \frac{q E_N(q) E_\pi(q) E_M(k)}{M^2} \mid \frac{h_{\Delta \leftrightarrow \pi N}(q)}{E - E_N(k) - E_\Delta(k) - \Sigma(k, E)} \mid^2 \,. \tag{38}$$

To have a better understanding of Eq. (36), we use Eq. (25) to change the integration variable

$$dk = -\frac{M E_N(k)}{kE} dM \,. \tag{39}$$

Eq. (36) can then be written as an integral over the invariant mass M of the πN subsystem

$$d\sigma^J_{L_\Delta S_\Delta, LS} = \frac{4\pi}{k_0^2}\left(J + \frac{1}{2}\right)\int_{m_\pi + m_N}^{E - m_N} dM \mid \Gamma_{N\Delta}(M)^{1/2} T^J_{L_\Delta S_\Delta, LS}(k, k_0, E) \rho^{1/2}_{NN}(E) \mid^2 \,. \tag{40}$$

where

$$\Gamma_{N\Delta}(M) = \pi \left[\frac{k E_N(k) E_M(k)}{E}\right]\left[\frac{q E_N(q) E_\pi(q)}{M}\right]$$
$$\times \mid \frac{h_{\Delta,\pi N}(q)}{E - E_N(k) - E_\Delta(k) - \Sigma(k, E)} \mid^2 \,. \tag{41}$$

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Note that in evaluating the above integral, we need to use Eqs. (26) and (27) to express k and q in terms of the invariant mass M and the total energy E. By Eqs. (26) and (34), the self-energy $\Sigma(k, E)$ is also a function of the total energy E and the invariant mass M.

Equation (40) is an exact expression. The integrand contains the dependence on the invariant mass M of the Δ^*, $d\sigma^J_{L_\Delta S_\Delta, LS}/dM$, which is needed for a detailed transport-equation calculation. For practical applications and making contact with the current calculations of relativistic heavy-ion collisions, it is interesting to develop an averaging procedure such that the cross section for a given total energy E can be written in terms of only one averaged Δ^*. This is our next task.

We use Eqs. (26) and (41) to define an average relative momentum $\bar{k}$ of the $N\Delta^*$ two-body system

$$\bar{k} = \frac{\int_{m_N+m_\pi}^{E-m_N} \Gamma_{N\Delta}(M) k \, dM}{\int_{m_N+m_\pi}^{E-m_N} \Gamma_{N\Delta}(M) dM} \, , \tag{42}$$

The corresponding average mass of Δ^* is then calculated from Eq. (25)

$$\bar{M}^2 = E^2 - 2EE_N(\bar{k}) + m_N^2 \, . \tag{43}$$

Assuming that the transition t-matrix in the integrand of Eq. (40) can be calculated at the average momentum $\bar{k}$, we then obtain the following "factorized" form

$$\sigma^{tot}_{N\Delta^*\leftarrow NN} = \sum_{J,L_\Delta S_\Delta, LS} d\bar{\sigma}^J_{L_\Delta S_\Delta, LS} \, , \tag{44}$$

with

$$d\bar{\sigma}^J_{L_\Delta S_\Delta, LS} = \frac{4\pi}{k_0^2}(J + \frac{1}{2}) \mid \rho^{1/2}_{N\Delta}(E) T^J_{L_\Delta S_\Delta, LS}(\bar{k}, k_0, E) \rho^{1/2}_{NN}(E) \mid^2 \, . \tag{45}$$

Here we have introduced a phase-space factor for the $N\Delta^*$ two-body state

$$\rho_{N\Delta}(E) = \int_{m_N+m_\pi}^{E-m_N} dM \Gamma_{N\Delta}(M) \, . \tag{46}$$

Equation (45) has the form similar to that introduced in the literature [4,5]. Here we have explicitly shown how it can be derived from the Hamiltonian formulation [11–13] of the Δ

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excitation by using the "factorization approximation". ΞΞThe accuracy of this approxima-
tion will be examined in section V. We emphasize here that that $\Gamma_{N\Delta}(M)$, defined in Eq.
(41), is the consequence of a unitary formulation of the πNN processes listed in Eqs. (1)-(3).
Equation (46) is significantly different from the parameterizations of the $N\Delta$ phase-space
introduced in Refs. [4,5].

We note that Eq. (45) has the usual form for the binary collisions involving only stable
particles. For example, the exact expression for $NN \rightarrow NN$ process can be obtained by
replacing $\rho_{N\Delta}(E)$ in Eq. (45) by $\rho_{NN}(E)$.

By using the $N\Delta^*$ representation of the πNN state, it is straightforward to carry out
similar derivations to express the $\pi NN \rightarrow NN$ and $\pi NN \rightarrow \pi NN$ cross sections in terms
of $N\Delta^* \rightarrow NN$ and $N\Delta^* \rightarrow N\Delta^*$ cross sections. If the averaging procedure defined in Eqs.
(42) and (46) is also applied, we obtain the following expressions

$$\sigma^{tot}_{NN \leftarrow N\Delta^*} = \sum_{J,LS,L_\Delta S_\Delta} d\bar{\sigma}^J_{LS,L_\Delta S_\Delta} \tag{47}$$

with

$$d\bar{\sigma}^J_{LS,L_\Delta S_\Delta} = \frac{4\pi}{\bar{k}^2} \mid \rho^{1/2}_{NN}(E) T^J_{LS,L_\Delta S_\Delta}(k_0,\bar{k},E)\rho^{1/2}_{N\Delta}(E) \mid^2 , \tag{48}$$

for the $N\Delta^* \rightarrow NN$ reaction, and

$$\sigma^{tot}_{N\Delta^* \leftarrow N\Delta^*} = \sum_{J,L'_\Delta S'_\Delta,L_\Delta S_\Delta} d\bar{\sigma}^J_{L'_\Delta S'_\Delta,L_\Delta S_\Delta} , \tag{49}$$

with

$$d\bar{\sigma}^J_{L'_\Delta S'_\Delta,L_\Delta S_\Delta} = \frac{4\pi}{\bar{k}^2}(J + \frac{1}{2}) \mid \rho^{1/2}_{N\Delta}(E) T^J_{L'_\Delta S'_\Delta,L_\Delta S_\Delta}(\bar{k},\bar{k},E)\rho^{1/2}_{N\Delta}(E) \mid^2 , \tag{50}$$

for the $N\Delta^* \rightarrow N\Delta^*$ reaction.

We can further extend the above formula to calculate the differential cross sections
involving a Δ^* resonance in either the entrance channel or the exit channel. Specifically, we
can define a general scattering amplitude

$$\tilde{T}_{\alpha\beta}(\vec{k}_\alpha, \vec{k}_\beta, E) = \sum_{JM_J, L_\alpha S_\alpha, L_\eta S_\beta} y_{L_\alpha S_\alpha}^{JM_J}(\hat{k}_\alpha) y_{L_\beta S_\beta}^{JM_J^+}(\hat{k}_\beta)$$

$$\times \rho_\alpha^{1/2}(E) T_{L_\alpha S_\alpha, L_\beta S_\beta}^J(\bar{k}_\alpha, \bar{k}_\beta, E) \rho_\beta^{1/2}(E), \qquad (51)$$

where α, β can be NN or $N\Delta^*$ states, and $|\vec{k}_\alpha| = \bar{k}, k_0$ for $\alpha = N\Delta^*, NN$. The differential cross sections are then of the following form

$$\frac{d\sigma_{\alpha\beta}(E)}{d\Omega} = \frac{16\pi^2}{\bar{k}_\beta^2} \frac{1}{(2s_{1_\beta}+1)(2s_{2_\beta}+1)} \sum_{spins} |< m_{1_\alpha} m_{2_\alpha} \mid \tilde{T}_{\alpha\beta}(\vec{k}_\alpha, \vec{k}_\beta, E) \mid m_{1_\beta} m_{2_\beta} >|^2 \qquad (52)$$

It is easy to show that the total cross sections Eqs. (44), (47), and (49) can be obtained by integrating Eq. (52) over the scattering angle Ω.

IV. MEDIUM EFFECTS ON THE Δ PROPAGATION

We follow the approach developed in Ref. [14] to calculate the medium effects on the Δ propagation. The resulting Δ potential is then used to calculate the medium effects on the scattering cross sections defined in the previous section.

In the one-hole-line approximation, the propagator of a Δ with momentum $\vec{p}_\Delta$ and energy ϵ_Δ in nuclear matter can be written as

$$G_\Delta(\vec{p}_\Delta, \epsilon_\Delta) = \frac{1}{\epsilon_\Delta - E_\Delta(\vec{p}_\Delta) - \Sigma^{(1)}(\vec{p}_\Delta, \epsilon_\Delta) - \Sigma^{(2)}(\vec{p}_\Delta, \epsilon_\Delta)}, \qquad (53)$$

with

$$\Sigma^{(1)}(\vec{p}_\Delta, \epsilon_\Delta) = \int \frac{Q_{\pi N}(k, p_\Delta) \mid h_{\pi N \leftrightarrow \Delta}(k) \mid^2 k^2 dk}{\epsilon_\Delta - \sqrt{(E_N(k) + E_\pi(k))^2 + p_\Delta^2} + i\epsilon}, \qquad (54)$$

where $Q_{\pi N}(k, p_\Delta)$ is an angle-average Pauli operator [14] for the πN state. The medium effect on the Δ propagation is calculated from the $N\Delta$ G-matrix

$$\Sigma^{(2)}(\epsilon_\Delta, \vec{p}_\Delta) = \int_{p \leq p_F} d\vec{p} < \vec{p}_\Delta \vec{p} \mid G_{N\Delta, N\Delta}(W = (\epsilon_\Delta + \epsilon_N(\vec{p}))) \mid \vec{p}_\Delta \vec{p} >, \qquad (55)$$

where p_F is the Fermi momentum of nuclear matter.

It is convenient to calculate the G matrix in partial-wave representation. For symmetric nuclear matter(the total isospin and the total angular momentum are both equal to zero), we find that

$$\Sigma^{(2)}(\vec{p}_\Delta, \epsilon_\Delta) = \frac{1}{(2s_\Delta + 1)(2\tau_\Delta + 1)} \sum_{L_\Delta S_\Delta JT} \frac{(2J+1)(2T+1)}{4\pi}$$

$$\times \int_{p \leq p_F} d\vec{p} \, G^{JT}_{L_\Delta S_\Delta, L_\Delta S_\Delta}(q, q, \omega(\epsilon_\Delta, \vec{p}_\Delta, \vec{p}), P) \,, \tag{56}$$

where q is the relative momentum, ω is the collision energy in the $N\Delta$ center of mass frame, and P is the total momentum. In the nonrelativistic baryon approximation, we have

$$P = |\, \vec{p}_\Delta + \vec{p} \,| \,,$$

$$q = |\, \frac{m_\Delta^0 \vec{p} - m_N \vec{p}_\Delta}{m_\Delta^0 + m_N} \,| \,,$$

$$\omega(\epsilon_\Delta, \vec{p}_\Delta, \vec{p}) = \epsilon_\Delta + \epsilon_N(\vec{p}) - m_\Delta^0 - m_N - \frac{(\vec{p}_\Delta + \vec{p})^2}{2(m_\Delta^0 + m_N)} \,,$$

where $m_\Delta^0 = 1280$ MeV is the bare mass of the Δ determined from fitting the πN phase shifts in P_{33} channel.

The equations for the G-matrix are identical to Eqs. (6)-(9) except that the propagators are modified. In the partial-wave representation, each equation has the following structure

$$G^{JT}_{\alpha,\delta}(q', q, \omega, P) = V^{JT}_{\alpha,\delta}(q', q, \omega)$$

$$+ \sum_\gamma \int q''^2 dq'' \frac{V^{JT}_{\alpha,\gamma}(q', q'', \omega) Q_\gamma(q'', P, p_F)}{\omega - W_\gamma(q'', P) + i\epsilon} G^{JT}_{\gamma,\delta}(q'', q, \omega, P) \tag{57}$$

where the greek subindices denote collectively the particle channels, NN or $N\Delta$, and the orbital and spin quantum numbers. In the propagator of Eq. (57), $Q_\gamma(q'', P, p_F)$ is the angle-average Pauli operator [14] and

$$W_\gamma(q, P) = \epsilon_N(\bar{p}) + \epsilon_N(\bar{p}) \tag{58}$$

for $\gamma = NN$ channel, and

$$W_\gamma(q) = \epsilon_N(\bar{p}_N) + \epsilon_\Delta(\bar{p}_\Delta) \tag{59}$$

for $\gamma = N\Delta$ channel. The single particle energies in the above equations are calculated by using the angle-average momenta

$$\bar{p} = \sqrt{q^2 + \frac{1}{4}P^2},$$

and

$$\bar{p}_N = \sqrt{(\frac{m_N P}{m_\Delta^0 + m_N})^2 + q^2},$$

$$\bar{p}_\Delta = \sqrt{(\frac{m_\Delta P}{m_\Delta^0 + m_N})^2 + q^2}.$$

The nucleon single particle energy $\epsilon_N(p)$ is taken from a variational many-body calculation by Wiringa [21]. The single particle energy for the Δ is determined by the following self-consistency condition

$$Re(\epsilon_\Delta(\vec{p}_\Delta)) = m_\Delta^0 + \frac{p_\Delta^2}{2m_\Delta^0} + Re(\Sigma^{(1)}(\vec{p}_\Delta, \epsilon_\Delta(\vec{p}_\Delta)) + Re(U_\Delta(\vec{p}_\Delta))), \tag{60}$$

where we have defined the Δ potential

$$U_\Delta(\vec{p}_\Delta) = \Sigma^{(2)}(\vec{p}_\Delta, \epsilon_\Delta(\vec{p}_\Delta)). \tag{61}$$

The medium effects on the cross sections of binary collisions defined in section III can be included by replacing the scattering t-matrix in Eqs. (45), (48), (50), and (51) by the G-matrix. The collision energy is calculated from real parts of the single particle energies $\epsilon_N(p)$ of Ref. [21] and $\epsilon_\Delta(p_\Delta)$ defined by the self consistent condition Eq. (60).

V. RESULTS AND DISCUSSIONS

We have applied the formula developed in section III to investigate the cross sections of binary collisions involving a Δ^* excitation in either the entrance channel or the exit channel. In Fig. 3, we show that the $NN \to N\Delta^*$ cross sections (dashed curve) calculated by using the factorized form Eqs. (44)-(45) can reproduce reasonably well the exact calculation of $NN \to \pi NN$ using Eqs. (35) and (40). This suggests that the factorized forms defined by Eqs. (44)-(52) are sufficient for investigating the main features of relativistic heavy-ion collisions. The calculated total cross sections for all processes involving Δ^* are compared in Fig. 4. They have quite different energy dependencies. The difference between $NN \to N\Delta^*$ and $N\Delta^* \to NN$ is due to the flux factors $1/k_0^2$ of Eq. (45) and $1/\bar{k}^2$ of Eq. (48). This can be understood from Fig. 5 in which the calculated $\bar{k}$ for the $N\Delta^*$ channel is compared with the

momentum k_0 of the NN channel. The calculated mass of the Δ^* is also displayed there. We see that as the collision energy increases the produced Δ^* becomes heavier and moves faster.

The predicted differential cross sections at several energies are compared in Fig. 6. We see that the angular distributions for $NN \to N\Delta^*$ is quite different from that of $N\Delta^* \to NN$. This is due to the fact that the important partial waves in NN and $N\Delta$ are quite different. For example, in the J=2 channel the outgoing particles are in $^1D_2$ for the $N\Delta^* \to NN$, but are in $^5S_2$ for the $NN \to N\Delta$. Clearly such a dynamical difference in differential cross sections can not be easily obtained by using detailed balance [3–5] to relate $N\Delta^* \to NN$ to the data of the $NN \to NN\pi$ reaction. We further notice that the predicted $N\Delta^* \to N\Delta^*$ differential cross sections are forward peaked. This is also quite different from the usual assumption that $N\Delta^* \to N\Delta^*$ is identical to $NN \to NN$ apart from some isospin factors. The results shown in Figs. 4 and 6 are the consequences of the considered meson-exchange mechanisms.

To get some ideas about the medium effects on collision cross sections, we assume that the mean field effect on the Δ propagation is the potential U_Δ defined in Eqs. (56) and (61). The calculated U_Δ at several densities are displayed in Fig. 7. Both the real parts and imaginary parts depend strongly on the density and the momentum. Qualitatively, our results indicate that the Δ moves almost freely in nuclear medium at high momentum, but is slowed down considerably and is easily annihilated at low momentum. It will be interesting to explore the consequence of the predicted U_Δ in determining the pion yields in relativistic heavy-ion collisions.

The predicted $U_\Delta(p)$ is used to evaluate the Δ single particle energy Eq. (60) and the G matrix elements which are the inputs to the calculations of the cross sections in medium, as explained in section IV. We have found that the medium effects are most dramatic in the region where the collision energies are close to the πNN threshold in free space. This can be illustrated in a calculation where a Δ^* with a mass of 1236 MeV propagates under the influence of the mean field U_Δ and collides with a nucleon at rest in nuclear matter.

The predicted dependence of the $N\Delta^* \to NN$ and $N\Delta^* \to N\Delta^*$ total cross sections on the density and the momentum is displayed in Fig. 8. There are two medium effects. The first one is the mean field effect on the propagators of the G-matrix in Eq. (57). This effect is found to be not very large in the momentum region considered. The main medium effect is due to the change of the collision energies by the Δ potential U_Δ in the entrance channels. Since the potential energy(real part) becomes more attractive as density increases (Fig. 7), the collision energy for a given momentum of the incident Δ is shifted to the lower energy region closer to the pion production threshold where the cross sections vary rapidly as shown in Fig. 4. We therefore see in Fig. 8 that the $N\Delta^*$ collision cross sections in the low momentum region depend strongly on the density, and are greatly suppressed at high densities. A similar situation is also found for $NN \to N\Delta^*$, as illustrated in the lower half of Fig. 9. The corresponding medium effects on the $NN \to NN$ are less dramatic since the NN elastic cross sections do not vary so rapidly in the energy region near pion production threshold.

VI. SUMMARY

The binary collisions involving a Δ resonance in either the entrance channel or the exit channel have been investigated within a Hamiltonian formulation of πNN interactions [11–13]. An averaging procedure has been developed to define the experimentally measured $NN \to \pi NN$ cross section in terms of an effective $NN \to N\Delta^*$ cross section. In contrast to previous works [4,5], the main feature of the present approach is that the mass and the momentum of the produced Δ^* at each collision energy are calculated dynamically from the bare $\Delta \leftrightarrow \pi N$ vertex interaction of the model Hamiltonian and are constrained by the unitarity condition. The procedure is then extended to define the effective cross sections for the experimentally inaccessible $N\Delta^* \to NN$ and $N\Delta^* \to N\Delta^*$ reactions. The calculations have been performed to predict the energy dependencies and angular distributions of these cross sections.

The Δ mean field in nuclear matter has been calculated by using the Bruckner-Hartree-Fock approximation developed in [14]. It is found that the Δ moves almost freely at high momenta, but is slowed down considerably and get easily absorbed at low momenta. By including the medium effect on the Δ propagation, the dependence of the effective cross sections of the $N\Delta^*$ collisions on the density has been investigated. It is demonstrated that the density dependence is most dramatic in the energy region close to the pion production threshold.

Our results can be used in the relativistic heavy-ion calculations using transport equations [3–9]. To make further progress in investigating Δ-rich matter, it is necessary to investigate the $\Delta^*\Delta^*$ collisions and the effects due to the higher mass N^* resonances. This requires the extension of the πNN model Hamiltonian defined by Eqs. (4)-(5) to include the two$-\pi$ channels and the development of a scattering theory with $\pi\pi NN$ unitary condition. Such a Hamiltonian model can be accurately constructed only when the dynamics of our present understanding of the N^* excitations in πN and γN reactions is improved quantitatively. The proposed pion facility at GSI [2] and the N^* program at CEBAF are essential for making progress in this direction.

REFERENCES

[1] U. Mosel and V. Metag, Nucl. Phys. News **3** (1993) 25.

[2] R. Averbeck, A. Döppenschmidt, V. Metag, and R. S. Simon, GSI-Nachrichten $\mathbf{10-95}$ (1995) 15.

[3] G. F. Bertsch and S. Das Gupta, Physics Report **160** (1988) 189.

[4] P. Danielewicz and G. F. Bertsch, Nucl. Phys. **A533** (1991) 712.

[5] Gy. Wolf, B.Batko, W. Cassing, U. Mosel, K. Niita, and M. Schäfer, Nucl. Phys. **517** (1990) 615.

[6] Gy. Wolf, W. Cassing, and U. Mosel, Nucl. Phys. **A552** (1993) 549.

[7] B.A. Li and W. Bauer, Phys. Rev. **C44** (1991) 450.

[8] L. Xiong. Z.G. Wu, C.M. Ko, and J.Q. Wu, Nucl. Phys. **A512** (1990) 772.

[9] C. Muntz *et al.*, Z. Phys. **A352** (1995) 175.

[10] J. R. Jeukenne, A. Lejeune, and C. Mahaux, Phys. Repo. **25** (1976) 83.

[11] T.-S. H. Lee, Phys. Rev. Lett. **20** (1983) 1571; Phys. Rev. **C29** (1984) 195.

[12] T.-S. H. Lee and A. Matsuyama, Phys. Rev. **C32** (1985) 516; Phys. Rev. **C36** (1987) 1459.

[13] A. Matsuyama and T.-S. H. Lee, Phys. Rev. **C34** (1986) 1900; Nucl. Phys. **A526** (1991) 547.

[14] T.-S. H. Lee and K. Ohta, Phys. Rev. **C25** (1982) 3043.

[15] V. Carlson *et al.*, ANL-HEP-PR-95-64 preprint, Argonne National Laboratory, (1995).

[16] H. Garcilazo and T. Mizutani, πNN System, World Scientific (1990).

[17] M. Betz and T.-S. H. Lee, Phys. Rev. **C23** (1981) 375.

[18] G.-H. Niepleaus, M. Gari, and B. Sommer, Phys. Rev. **C20** (1979) 1096.

[19] M. Lacombe, B. Loiseau, J. M. Richard, R. Vinh Mau, J. Cott, P. Pine, and K. Tourreil, Phys. Rev. **C21** (1980) 861.

[20] H. Popping, P. U. Sauer, and X.Z. Zhang, Nucl. Phys. **A474** (1987) 557.

[21] R. B. Wiringa, Phys. Rev. **C38** (1988) 1010.

[22] B. J. VerWest and R. A. Arndt, Phys. Rev. **C25** (1982) 1979.

FIGURES

FIG. 1. The polarization observables C_{LL} and C_{LS} of np scattering predicted by the πNN model [11–13] are compared with the recent data [15].

FIG. 2. Graphical representation of the $NN \to N\Delta \to \pi NN$ reaction.

FIG. 3. The calculated $NN \to \pi NN$ (solid curve) and $NN \to N\Delta^*$ (dotted curve) total cross sections are compared. The data are from the compilation of Ref. [22].

FIG. 4. The predicted total cross sections for the binary collisions involving a Δ^* in either the entrance channel or the exit channel are compared.

FIG. 5. The mass $\bar{M}^*$ and momentum $\bar{k}$ of the Δ^* in the $NN \to N\Delta^*$ reaction. E_L is the incident nucleon energy in the laboratory frame.

FIG. 6. The predicted angular distributions evaluated at energies (in terms of the corresponding nucleon laboratory energies) E_L =400 MeV (dotted curves), 700 MeV (solid curves), and 900 MeV (dashed curves).

FIG. 7. The calculated Δ potentia $U_\Delta(p_\Delta)$ at several densities. The density ρ is in units of $\rho_0 = 0.16$ nucleons-fm^{-3}.

FIG. 8. The momentum-dependence of the calculated total cross sections of $N\Delta^* \to NN$ and $N\Delta^* \to N\Delta^*$ collisions in nuclear matter. The density ρ is in units of $\rho_0 = 0.16$ nucleons-fm^{-3}.

FIG. 9. The momentum-dependence of the calculated $NN \to NN$ and $NN \to N\Delta^*$ collisions in nuclear matter. The density ρ is in unit of $\rho_0 = 0.16$ nucleons-fm^{-3}.